International Journal of Theoretical Physics, Vol. 22, No. 4, 1983

Does the Electromagnetic Mass of an Electron Depend on Where It Is? [1]

P. W. Milonni [2]

Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701

Received June 26, 1981

The question of whether the electromagnetic mass of the electron depends upon its electromagnetic environment is discussed in connection with previous theoretical and experimental work. When the quantization of the field in other than free space is properly understood, it is evident that the true electromagnetic mass of the electron is unaltered.

Orthodox radiation theory leads to the conclusion that a part δm of the mass of an electron is of electromagnetic origin. It is also asserted that no experiment can differentiate between δm and the remaining, bare mass of the electron (see, for example, Schweber, 1962). Other radiation theories, in which the electron experiences no self-interaction and therefore has no electromagnetic mass, have been advanced (Wheeler and Feynman, 1945). It is therefore of interest to consider the possibility that δm may be affected by the presence of conducting plates (Power, 1966; Guttrich and Billman, 1967; Golub and Guttrich, 1967).

I recently considered the possibility, in light of a previous interest in the modification of spontaneous emission rates by the presence of conducting plates (Milonni and Knight, 1973), that this modification of δm might provide a test of the reality of electromagnetic mass. Barton's careful work on quantum electrodynamics near conducting plates (Barton, 1974) brought the work of Power (1966) to my attention, and so I learned that the modification of δm by conducting plates was not a new idea. However, I now believe that the proposed experiment does not actually involve a modification of δm *per se*, so that the orthodox view of the unobservability of electromagnetic mass is not in question.

[1] Research supported in part by dual funding by NSF-EPSCOR Grant ISP 8011447 and the State of Arkansas.

[2] Temporary address (1982): Theoretical Division (T-12), Los Alamos National Laboratory, Los Alamos, New Mexico 87545.

0020-7748/83/0400-0323$03.00/0

Consider first an electron in free, unbounded space. The second-order, nonrelativistic result for the electromagnetic self-energy of the point electron due to the $\mathbf{A}\cdot\mathbf{p}$ perturbation is

$$\Delta E^{(0)} = \frac{-4e^2\Omega}{3\pi mc^3}\frac{\mathbf{p}^2}{2m} \tag{1}$$

where Ω is a high-frequency cutoff. The radiation reaction field of the point electron is

$$\mathbf{E}_{RR}^{(0)} = \frac{2e}{3c^3}\dddot{\mathbf{r}} - \frac{4e\Omega}{3\pi c^3}\ddot{r} \tag{2}$$

for the same cutoff Ω, thus implying an electromagnetic mass δm given by

$$\delta m/m = \frac{4e^2\Omega}{3\pi mc^3} \tag{3}$$

This is the same as the coefficient multiplying $\mathbf{p}^2/2m$ in equation (1), so that the self-energy $\Delta E^{(0)}$ is just the correction to the bare electron energy arising from the addition of δm to the bare mass $m - \delta m$.

Now consider an electron at a distance $z > 0$ from a perfectly conducting plate at $z = 0$. The Coulomb-gauge vector potential satisfying the boundary conditions in the right half-space may be written as (Barton, 1974)

$$\mathbf{A}(\mathbf{x}) = \left(\frac{\hbar c^2}{\pi^2}\right)^{1/2}\int_0^\infty dl \int_{-\infty}^\infty d^2k\,\omega^{-1/2}$$
$$\times\left\{a_1(\mathbf{k},l)\hat{k}\times\hat{z}\sin lz + a_2(\mathbf{k},l)\left[\frac{ilc}{\omega}\hat{k}\sin lz - \frac{kc}{\omega}\hat{z}\cos lz\right]\right\}$$
$$\times e^{i\vec{\mathbf{k}}\cdot\vec{\mathbf{x}}} + \text{h.c. (hermitian conjugate)} \tag{4}$$

where the $a_s(\mathbf{k},l)$ are photon annihilation operators, $\omega^2/c^2 = k^2 + l^2$, and $\mathbf{k} = k\hat{\mathbf{k}}$ is orthogonal to $\hat{z}$. Using this mode expansion in the second-order expression for the electron self-energy arising from the $\mathbf{A}\cdot\mathbf{p}$ term, we obtain after a tedious calculation

$$\Delta E = \Delta E^{(0)} + \mathbf{p}_{\|}^2\left(\frac{e}{2mc}\right)^2\Big/2z \tag{5}$$

if it is assumed, to avoid irrelevant complications, that the electron velocity is parallel to the wall. The subscript on the operator **p** in equation (5) is used to remind us of this assumption.

If the z-dependent portion of this self-energy is associated with a modification of δm due to the plate, then it should be possible to calculate it from the radiation reaction field, as in the case of an electron in free space. Again using the mode expansion (4), we obtain the following expression for the leading term (in z^{-1}) in the radiation reaction field of the electron in the half-space $z>0$:

$$\mathbf{E}_{\mathrm{RR}} = \mathbf{E}_{\mathrm{RR}}^{(0)} - \frac{e}{4c^2 z}\dot{\mathbf{v}}_{\parallel}\left(t - \frac{2z}{c}\right) \tag{6}$$

again ignoring any motion toward or away from the plate. The second term in (6) is the only z-dependent part of $\mathbf{E}_{\mathrm{RR}}$ involving the electron acceleration, and is therefore the term of interest here.

The nonrelativistic equation of motion for an electron constrained to move in the x direction only is therefore

$$\ddot{x}(t) - \gamma\dddot{x}(t) + \alpha\dddot{x}\left(t - \frac{2z}{c}\right)U\left(t - \frac{2z}{c}\right) = \frac{1}{m}F(t) \tag{7}$$

where $\gamma = 2e^2/3mc^3$, $\alpha = e^2/4mc^2z$, U is the unit step function, and $F(t)$ is a prescribed applied force turned on at $t=0$. The solution for the acceleration $a(t) = \ddot{x}(t)$ is

$$a(t) = e^{t/\gamma}\sum_{n=0}\frac{1}{n!}\left(\frac{-3c}{8z}\right)^n U\left(t - \frac{2nz}{c}\right)e^{-2nz/c\gamma}$$

$$\times\left[a(0)\left(t - \frac{2nz}{c}\right)^n - \frac{3c^3}{2e^2}\int_0^{t-2nz/c} dt_1 F(t_1)\left(t - t_1 - \frac{2nz}{c}\right)^n e^{-t_1/\gamma}\right] \tag{8}$$

For times $t < 2z/c$ the electron motion is not affected by the plate:

$$a(t) = e^{t/\gamma}\left[a(0) - \frac{3c^3}{2e^2}\int_0^t dt_1 F(t_1)e^{-t_1/\gamma}\right], \qquad t < \frac{2z}{c} \tag{9}$$

The electron first feels the presence of the plate at time $t = 2z/c$, and, at further integral multiples of this signaling time, additional contributions to the electron acceleration appear. For very large times we have

$$a(t)=e^{t/\gamma}\sum_{n=0}^{\infty}\frac{1}{n!}\left(\frac{-3c}{8z}\right)^n$$

$$\times\left[a(0)t^n-\left(\frac{3c^3}{2e^2}\right)\int_0^t dt_1 F(t_1)(t-t_1)^n e^{-t_1/\gamma}\right]$$

$$=e^{t/\gamma'}\left[a(0)-\left(\frac{3c^3}{2e^2}\right)\int_0^t dt_1 F(t_1)e^{-t_1/\gamma'}\right] \tag{10}$$

where

$$\frac{1}{\gamma'}=\frac{1}{\gamma}-\frac{3c}{8z} \tag{11}$$

In order to avoid a "runaway" solution we must impose the acausal condition (see, for example, Plass, 1961)

$$a(0)=\frac{3c^3}{2e^2}\int_0^{\infty}dt_1 F(t_1)e^{-t_1/\gamma'} \tag{12}$$

on the initial acceleration, so that

$$a(t)=\frac{3c^3}{2e^2}\int_0^{\infty}dt_1 F(t+t_1)e^{-t_1/\gamma'} \tag{13}$$

for times t much greater than the signal time $2z/c$. In the absence of the plate we obtain equation (13) with γ' replaced by γ. Now

$$\frac{1}{\gamma'}=\frac{3c^3}{2e^2}[m+\delta m(z)]=\frac{1}{\gamma}\left[1+\frac{\delta m(z)}{m}\right] \tag{14}$$

where

$$\delta m(z)=-e^2/4c^2 z \tag{15}$$

This is just the correction to the electromagnetic mass that would be obtained by setting the signal time $2z/c$ equal to zero. It is also seen that the z-dependent correction to the self-energy (5) may be written as

$$p_{\parallel}^2\left(\frac{e}{2mc}\right)^2\Big/2z=-\frac{p_{\parallel}^2}{2m}\frac{\delta m(z)}{m} \tag{16}$$

Thus it appears that the electromagnetic mass of the electron is changed by the amount $\delta m(z)$ due to the presence of the plate.

But the solution (8) shows that things are not that simple. The electron first feels the presence of the plate at $t = 2z/c$. At $t = 4z/c$ there appears another correction to the electron's motion due to the presence of the plate. Additional corrections occur at successive integral multiples of the "photon bounce time" $2z/c$. Only after a time long enough for further "bounces" to be inconsequential does it make sense to say [equation (15)] that the electromagnetic mass of the electron has been modified by the presence of the plate. Then the electron motion [equation (13)] is *as if* its mass were modified by the presence of the plate. The result in the long-time limit can be understood in terms of an instantaneous interaction of the electron with the "acceleration field" of its image. But then the radiation reaction field acting on the electron is just the intrinsic, free-space radiation reaction field, $\mathbf{E}_{RR}^{(0)}$, plus the field reflected from the wall and acting back on the electron. Any valid radiation theory would predict that result, and so there is really no test here for the reality of electromagnetic mass.

The true electromagnetic mass of the electron determines (in part) its inertia *instantaneously*, and is not affected by a conducting plate or any other environmental characteristic. Any environmental modification of the electron motion must involve retardation, just as in the case of a conducting plate. Only on a time scale long compared to the signaling time between the electron and its environment does it appear that the electron responds to a force as if it has a mass different from its free-space observed mass. However, this modification of its mass can be understood as a result of the interaction of the electron with *other* charges.

In reality, of course, the electron motion in the simple example considered here cannot be constrained to motion in a single direction. The model considered here nevertheless elucidates the nature of the effect of the environment on the electron.

The z-dependent correction (16) to the electron "self"-energy is consistent with the standard external-field-type calculation in quantum electrodynamics. In the present case the radiative correction remaining after subtraction of the free-electron ($z \to \infty$) self-energy is just the far-field electron–image interaction. In general, however, it is not possible to associate such an interaction simply with a position-dependent electromagnetic mass (Barton, 1974). This is evident even in the simple example considered here, for if we retain more than the leading, "acceleration field" term in equation (6), there arise terms not associated with electromagnetic mass.

It should be emphasized also that the quantization of the field in the half-space leads automatically to retardation in the influence of the plate on the electron. At first thought it might seem that quantization of the field in mode functions appropriate to the space in question might already assume a long-term limit in which an electron at any point in the space "knows" the nature of its environment, the modes being determined by successive reflections of the field radiated by the electron. But this is not the case. The expansion of the field in terms of the appropriate mode functions fully accounts for retardation.

REFERENCES

S. S. Schweber, *An Introduction to Relativistic Quantum Field Theory* (Harper and Row, N.Y., 1962), pp. 527–8.

J. A. Wheeler and R. P. Feynman, *Rev. Mod. Phys.* **17**, 157 (1945).

E. A. Power, *Proc. Roy. Soc. Lond.* **A292**, 424 (1966).

G. L. Guttrich and K. W. Billman, *Phys. Lett.* **24A**, 53 (1967).

R. Golub and G. L. Guttrich, *Phys. Lett.* **24A**, 224 (1967).

P. W. Milonni and P. W. Knight, *Opt. Commun.* **9**, 119 (1973).

G. Barton, J. Phys. B: *Atom. Molec. Phys.* **7**, 2134 (1974).

G. N. Plass, *Rev. Mod. Phys.* **33**, 37 (1961).